

Preview

- There are several baseband digital modulation and demodulation techniques
- Noise: additive white Gaussian noise channel.
 - Obinary pulse modulation
 - O non-binary modulation methods..

Binary Signal Transmission

- In a binary communication system, binary data consisting of a sequence of 0's and I's are
- transmitted by means of two signal waveforms, say,
 - o s0(t) and s1 (t).
- Then each bit is mapped into a corresponding signal waveform according to the rule

$$0 \rightarrow s_0(t), \qquad 0 \le t \le T_b$$

$$1 \rightarrow s_1(t), \qquad 0 \le t \le T_b$$

• bit time interval, $T_b = 1/R$, R data rate (bits per second)

Binary Signal Transmission

- the signal is transmitted through a channel and corrupted the addition of noise, denoted as n(t),
- This is a sample function of a white Gaussian process with power spectrum N₀/2 watts/hertz.
- such a channel is called an additive white Gaussian noise (AWGN) channel
- The received signal waveform is expressed as

$$r(t) = s_i(t) + n(t), i = 0, 1, 0 \le t \le T_b$$

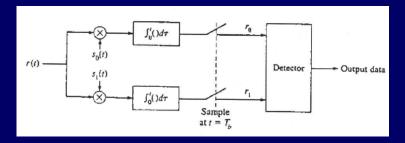
- The receiver must determine whether a 0 or a 1 was transmitted after observing the received signal r(t) in the interval o < r < T_b.
- The receiver must minimize the probability of error (optimum receiver).

Optimum Receiver

- Optimum Receiver for the AWGN Channel consists of two building blocks.
 - OEither signal correlator or matched filter
 - ODetector

Signal Correlator

 The signal correlator correlates the received signal r(t) with the two possible transmitted signals so(t) and s1(t), [Figure 1.]





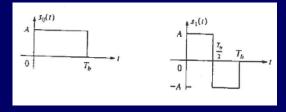
The signsl correlator computes the two outputs

$$r_0(t) = \int_0^t r(\tau) s_0(\tau) d\tau$$
$$r_1(t) = \int_0^t r(\tau) s_1(\tau) d\tau$$

• in the interval 0 < t < T_b

Illustrative-Problem 5.1 Determine the correlator outputs at the sampling instants.

 Suppose the signal waveforms s0(t) and s1(t) are the ones shown in Figure



 Let s0(t) be the transmitted signal, Then the received signal is

$$r(t) = s_0(t) + n(t), \qquad 0 \le t \le T_b$$

Determine the correlator outputs at the sampling instants.

when the signal r(t) is processed by the two signal correlator shown in Figure 1., the outputs r0 and r1 at the sampling instant t=T_b are

$$r_{0} = \int_{0}^{T_{h}} r(t)s_{0}(t) dt$$

$$= \int_{0}^{T_{h}} s_{0}^{2}(t) dt + \int_{0}^{T_{h}} n(t)s_{0}(t) dt$$

$$= \mathcal{E} + n_{0} \qquad \qquad \downarrow$$

$$n_{0}$$

$$r_{0} = \int_{0}^{T_{h}} r(t)s_{0}(t) dt$$

$$= \int_{0}^{T_{h}} s_{0}^{2}(t) dt + \int_{0}^{T_{h}} n(t)s_{0}(t) dt$$

$$= \mathcal{E} + n_{0}$$

$$r_{1} = \int_{0}^{T_{h}} r(t)s_{1}(t) dt$$

$$= \int_{0}^{T_{h}} s_{0}(t)s_{1}(t) dt + \int_{0}^{T_{h}} n(t)s_{1}(t) dt$$

$$= n_{1} \quad 0 \quad n_{1}$$

where n0 and n1 are the noise components at the output of the signal correlators



Note that the two signal waveforms are orthogonal,

$$\int_0^{T_h} s_0(t) s_1(t) \, dt = 0$$



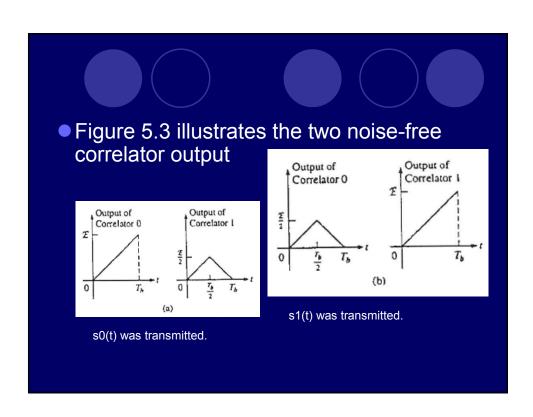
 when s1(t) is transmitted signal, the received signal must be

$$r(t) = s_1(t) + n(t), \qquad 0 \le t \le T_b$$

in this case, the signal correlator output are

$$r_0 = n_0$$

$$r_1 = \mathcal{E} + n_1$$



Matched Filter

- The matched filter provides an alternative to the signal correlator for demodulating the received signal r(t).
- A filter that is matched to the signal waveform s(t), 0 < t < T_b has an impulse response

$$h(t) = s(T_b - t), \qquad 0 \le t \le T_b$$

Matched Filter



$$y(t) = \int_0^t s(\tau)h(t-\tau)\,dt$$

$$y(t) = \int_0^t s(\tau)s(T_b - t + \tau) dt$$

$$y(T_h) = \int_0^{T_h} s^2(t) dt = \mathcal{Z}$$

- £ is the energy of the signal s(t).
- Therefore, the matched filter output at the sampling instant t = T_b is identical to the output of the signal correlator.

Illustrative-Problem 5.1

- Consider the use of matched filters for the demodulation of the two signal waveforms shown in Figure 5.2
- Determine the outputs.

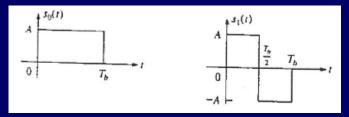


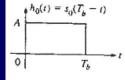
Figure 5.2

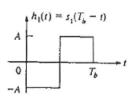
Illustrative-Problem 5.1

 The impulse responses of the two matched filters are

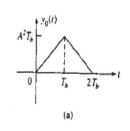
$$h_0(t) = s_0(T_b - t)$$

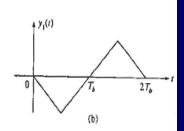
$$h_1(t) = s_1(T_b - t)$$





Illustrative-Problem 5.1





Signal outputs of matched filters when s0(t) is transmitted.

Detector



- The detector observes the correlator or matched filter outputs r0 and r1 and decides on whether the transmitted signal waveform is either s0(t) or s1(t), which correspond to tie
- transmission of either a 0 or a I, respectively, The aprimumderccbr is defined as thederector
- That minimizes the probability of error